Gauss' Divergence theorem

The theorem states that the surface integral of the normal component of a vector taken around a closed surface is equal to the integral of the divergence of the vector taken over the volume enclosed by the surface.

If V be the volume bounded by a closed surface S and $\vec{A}$ is a vector function of position with ~~contnuo~~ continuous derivatives, then according to the divergence theorem of Gauss'

$$\oiint \vec{A}\cdot\hat{n}\, ds = \iiint_V (\nabla\cdot\vec{A})\, dV$$

where $\hat{n}$ is the outward unit normal to S.

Gauss' divergence theorem thus expresses the relation between surface and volume integrals.

Stokes' theorem

The line integral of the tangential component of a vector taken around a simple closed curve is equal to the surface integral of the normal component of the curl of the vector taken over any surface having the curve as the boundary.

Mathematically, the theorem states that if S is an open two sided surface bounded by a simple closed curve C, and if $\vec{A}$ be any continuously differentiable vector point function, then

$$\oint_C \vec{A}\cdot d\vec{r} = \iint_S (\nabla\times\vec{A})\cdot\hat{n}\, ds = \iint_S (\nabla\times\vec{A})\cdot d\vec{s}$$

where the boundary is traversed in the positive direction. $\hat{n}$ is the outward drawn unit normal to the element of surface ds.

Stokes' theorem thus expresses the relation between line integral and surface integral of a vector.